

Surface zonal flows induced by thermal convection in a rapidly rotating thin spherical shell

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Outline

Background

Formulation and experiments setup

Results

Summary

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Background

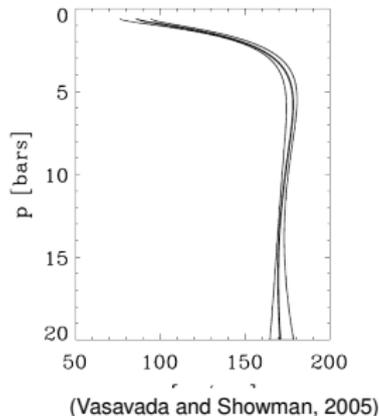
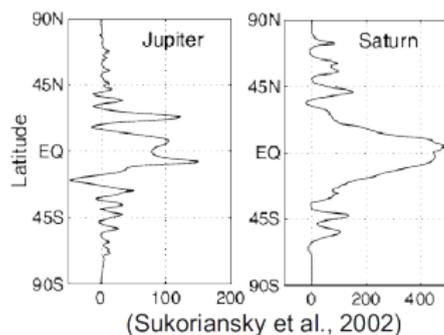
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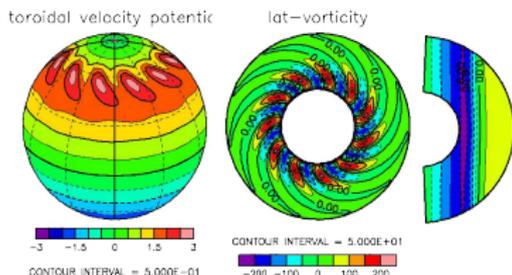
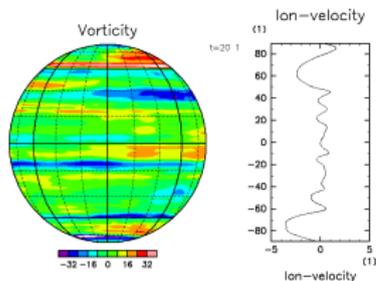
Surface flows of gas giant planets

- Surface flows of Jupiter and Saturn are characterized by the broad prograde jets around the equator and the narrow alternating jets in mid- and high-latitudes.
- It is not yet clear whether those surface jets are produced by convective motions in the “deep” region, or are the result of fluid motions in the “shallow” weather layer.



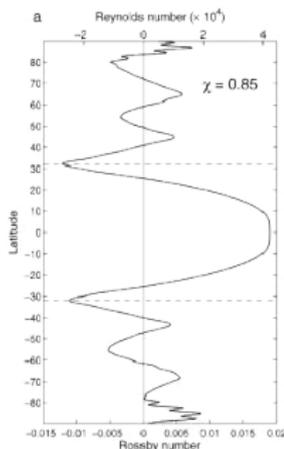
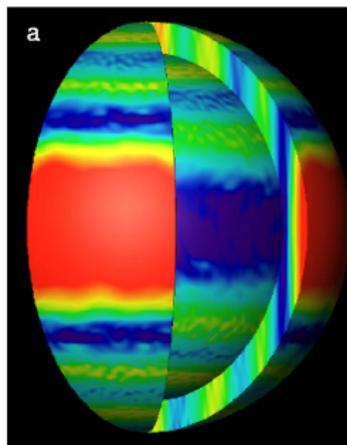
“Deep” models and “Shallow” models

- “Shallow” models
 - 2D turbulence on a rotating sphere
 - Primitive model
 - Result: Narrow alternating jets in mid- and high-latitudes.
 - Problem: the equatorial jets are not necessarily prograde
- “Deep” models
 - Convection in rotating spherical shells
 - Result: Produce equatorial prograde flows easily
 - Problem: difficult to generate alternating jets in mid- and high-latitudes



“Thin” spherical shell model

- Heimpel and Aurnou(2007) (hereafter, HA2007)
 - “Thin” spherical shell model with large Rayleigh number, small Ekman number.
 - Prograde jets and alternating jets in mid- and high-latitudes can produce simultaneously
 - However, **eight-fold symmetry** in the longitudinal direction is assumed.



Purpose

- HA2007: eight-fold symmetry in the longitudinal direction is assumed.
 - The artificial limitation of the computational domain may influence on the structure of the global flow field.
 - Zonal flows may not develop efficiently due to the insufficient upward cascade of two-dimensional turbulence
 - Stability of mean zonal flows may change with the domain size in the longitudinal direction.
 -
- In the present study:
 - Numerical simulations in the **whole** thin spherical shell domain.
 - Coarse spatial resolution and slow rotation rate are used due to the limit of computational resources.

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Model setup

- Boussinesq fluid in a rotating spherical shell.
 - scaling: the shell thickness, viscous diffusion time, temperature difference.

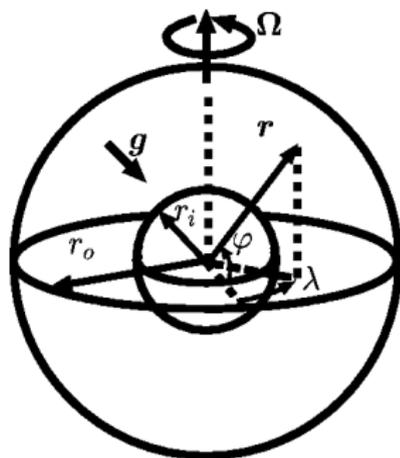
$$\nabla \cdot \mathbf{u} = 0,$$

$$E \left\{ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla^2 \mathbf{u} \right\} + 2\mathbf{k} \times \mathbf{u} + \nabla p = \frac{\text{Ra} E^2}{\text{Pr}} \frac{\mathbf{r}}{r_o} T,$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \frac{1}{\text{Pr}} \nabla^2 T.$$

- Parameters:

- Prandtl number: $\text{Pr} = \frac{\nu}{\kappa}$
- Rayleigh number: $\text{Ra} = \frac{\alpha g_o \Delta T D^3}{\kappa \nu}$
- Ekman number: $E = \frac{\nu}{\Omega D^2}$
- radius ratio: $\eta = \frac{r_i}{r_o}$



Experimental setup

- Boundary condition: Isothermal, Impermeable and Stress free.
- Input parameters:

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: η	0.75	0.85, 0.9
Ekman number: E	10^{-4}	10^{-6}
Modified Rayleigh number: Ra^*	0.05, 0.1	0.05

- the definition of modified Rayleigh number: $Ra^* = \frac{RaE^2}{Pr} = \frac{\alpha g \Delta T D}{\Omega^2 D}$
 - the ratio of Coriolis term and buoyancy term
- Output parameters:
 - (local) Reynolds number, Re , is equivalent to the non-dimensional velocity in the chosen scaling.
 - (local) Rossby number: $Ro = ERe$

Numerical methods

- Traditional spectral method.
 - Toroidal and Poloidal potentials of velocity are introduced.
 - The total wave number of spherical harmonics is truncated at 170, and the Chebychev polynomials are calculated up to the 48th degree.
 - The numbers of grid points:512, 256, and 48 in the longitudinal, latitudinal, and radial directions, respectively.
- In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies

$$\nu = \begin{cases} \nu_0, & \text{for } l \leq l_0, \\ \nu[1 + \varepsilon(l - l_0)^2], & \text{for } l > l_0. \end{cases}$$

- we choose $l_0 = 85, \varepsilon = 10^{-2}$
- The time integration is performed using the Crank-Nicolson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

Outline

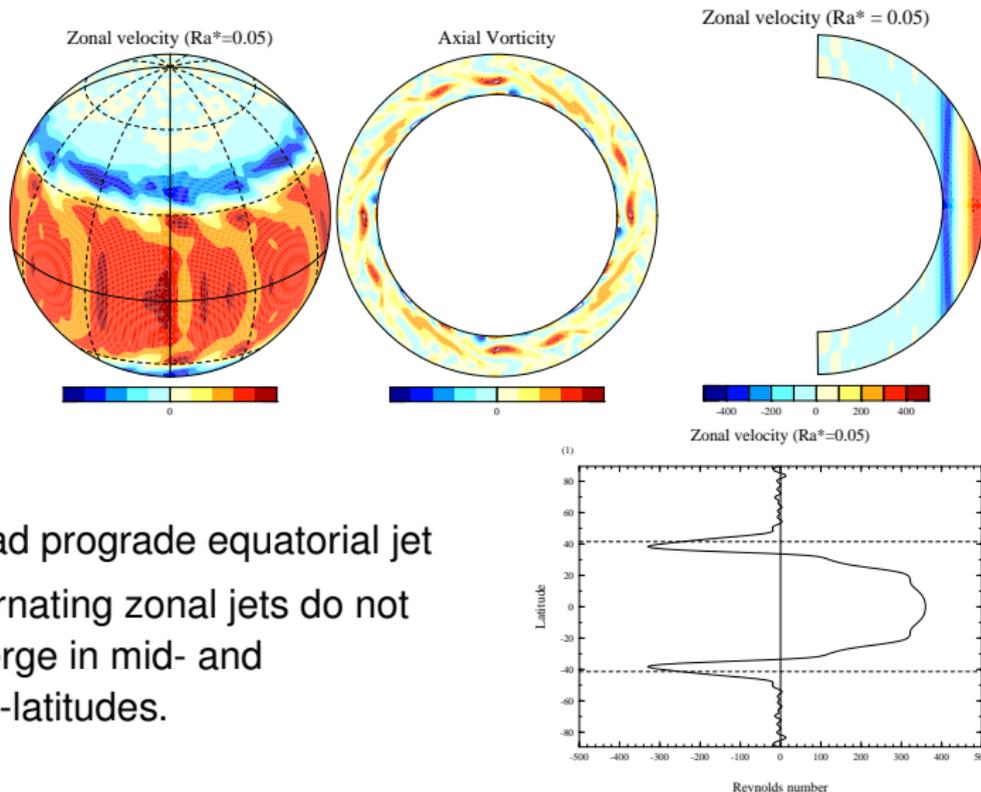
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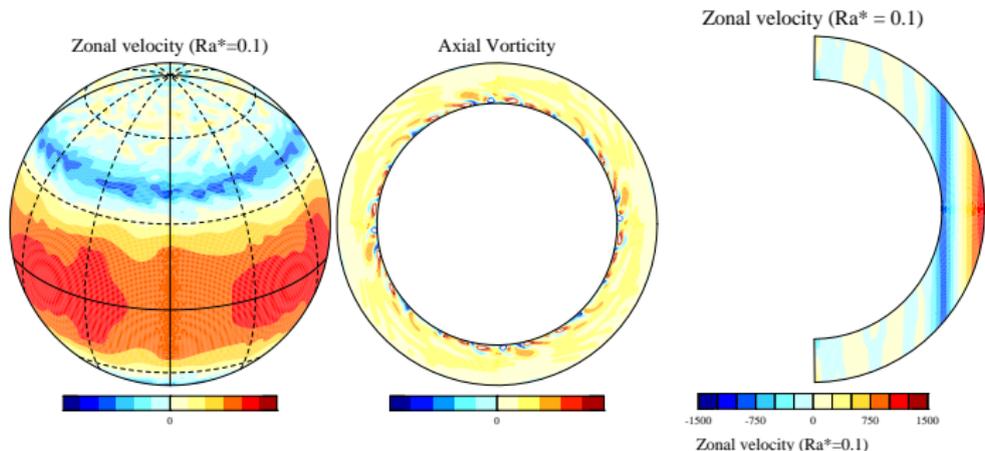
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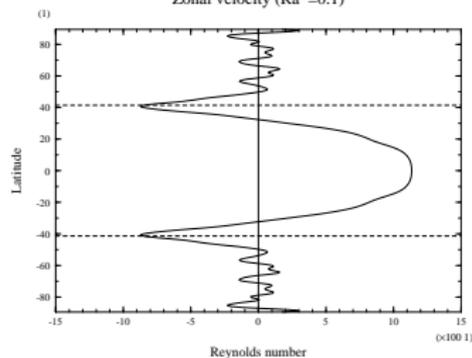
Results: $Ra^* = 0.05$



Results: $Ra^* = 0.1$



- Broad prograde equatorial jet
- Mean zonal flow: alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?



Results: output parameters

- Input parameters

parameters	present study	HA2007
Prandtl number: Pr	0.1	0.1
Radius ratio: η	0.75	0.85, 0.9
Ekman number: E	10^{-4}	10^{-6}
Modified Rayleigh number: Ra^*	0.05, 0.1	0.05

- Output parameters

parameters	present study	HA2007
local Reynolds number: Re	$3.59 \times 10^2, 1.13 \times 10^3$	5×10^4
local Rossby number: Ro	$3.59 \times 10^{-2}, 1.13 \times 10^{-1}$	$1.2 \times 10^{-2}, 2.5 \times 10^{-2}$

- small Re, i.e. weak jet
- Ro , i.e. slow rotation rate

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Summary and discussion

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- the profile of mean zonal flow:
 - Broad prograde equatorial jet
 - Alternating zonal jets emerge in mid- and high-latitudes
- Outer surface: alternating zonal jets in high latitudes ?
 - thick shell? , small Ra^* ?, large E ?
 - hyperdiffusivity?

In the future...

- More 'thin', 'fast rotating' spherical shell convection
 - $\eta = 0.75$ $\eta = 0.8, 0.85, 0.9$
 - $E = 10^{-4} \rightarrow E = 3 \times 10^{-5}$
- Investigation of generation mechanism
 - Comparison with the Rhines scale